

# Strings, T-duality breaking, and nonlocality without the shortest distance

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Received: 20 September 2006 / Revised version: 15 November 2006 /  
Published online: 26 January 2007 – © Springer-Verlag / Società Italiana di Fisica 2007

**Abstract.** T-duality of string theory suggests nonlocality manifested at the shortest possible distance. As an alternative, we suggest a nonlocal formulation of string theory that breaks T-duality at the fundamental level and does not require the shortest possible distance. Instead, the string has an objective shape in spacetime at all length scales, but different parts of the string interact in a nonlocal Bohmian manner.

**PACS.** 11.25.-w; 03.65.Ta

## 1 Introduction

T-duality [1–3] is an intriguing property of string theory. In its simplest form, T-duality is the remarkable fact that observable properties (such as the spectrum of the mass operator) of a closed string compactified on a circle with a radius  $R$  cannot be distinguished from that of a closed string compactified on a circle with the dual radius  $\alpha'/R$  (where  $\sqrt{\alpha'} = l_s$  is the fundamental length scale of string theory). Effectively,  $l_s$  turns out to be the shortest possible spatial distance, because a shorter string can always be reinterpreted as a longer string in the dual theory. The existence of the shortest possible distance suggests that a more fundamental formulation of string theory must be explicitly nonlocal at short distances [4]. Another suggestion that string theory is fundamentally nonlocal [4, 5] comes from the AdS/CFT correspondence [6], because this correspondence demonstrates a holographic property of strings, i.e., the property that observable properties of strings on an anti-de Sitter space can be described by a theory that lives only on the boundary of this space. However, the nonlocal nature of strings at the fundamental level is still far from being completely understood.

To understand the holographic principle and physics at the Planck scale at a more fundamental level, 't Hooft argues [7–9] that quantum mechanics (QM) should be formulated in terms of *deterministic hidden variables*, according to which QM is probabilistic only on the phenomenological level, whereas the more fundamental hidden (i.e., not observable with present technology) degrees of freedom satisfy deterministic laws of motion. Since, owing to the Bell theorem [10], QM does not allow local hidden variables, the hidden variables must necessarily be nonlocal.

Smolin argued [11] that matrix models, which give a non-perturbative formulation of string theory, might also be interpreted as nonlocal hidden variables. The best known and most successful hidden-variable formulation of quantum particles and fields is the *Bohmian* interpretation [12–19]. A version of the Bohmian interpretation of relativistic particles might be indirectly testable even with near-future technology [20]. The Bohmian interpretation has also been proposed as a possible interpretation of noncommutative QM [21]. Using our results on the manifestly covariant canonical quantization of fields [22, 23], we have recently argued [24] that the natural formulation of strings compatible with the world-sheet covariance on the quantum level is, indeed, a formulation in terms of Bohmian deterministic hidden variables. All these results suggest that the Bohmian interpretation might provide a more fundamental description of string theory.

In this paper we explore some physical consequences of the Bohmian deterministic formulation of string theory. We observe that the formulation of strings in terms of Bohmian hidden variables breaks T-duality at the fundamental level of the hidden variables. Thus, from this fundamental point of view,  $l_s$  can no longer be interpreted as the shortest possible spatial distance. Nevertheless, it does not mean that locality of strings is restored. Instead, nonlocality reappears in another form: as nonlocality typical of any hidden-variable completion of QM, as required by the Bell theorem.

## 2 T-duality and hidden variables

Consider a closed bosonic string compactified on a circle in the 25th spatial direction. A string winding  $m$  times

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around the compactified direction satisfies

$$X^{25}(\sigma + 2\pi) = X^{25}(\sigma) + 2\pi mR. \quad (1)$$

The momentum in this direction is also quantized:

$$P^{25} = \frac{n}{R}. \quad (2)$$

The mass of the string turns out to be given by

$$M^2 = \frac{n^2}{R^2} + \frac{m^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}[N + \tilde{N} - 2], \quad (3)$$

where the term in the square brackets corresponds to the contributions from the oscillatory degrees of freedom. In (3), the transformation  $R \rightarrow \alpha'/R$  is equivalent to the exchange  $n \leftrightarrow m$ . Therefore, the spectrum of  $M$  is invariant under the T-duality transformation. Classically, one can always determine whether the compactification radius is  $R$  or  $\alpha'/R$ , simply by watching the shape of the string that satisfies (1). In contrast, in QM, if one observes  $P^{25}$  or  $M$ , which are described by quantum mechanical operators that do not commute with  $X^{25}$ , then one cannot observe  $X^{25}$ , and thus one cannot observe the shape of the string. According to the orthodox interpretation of QM, quantities that cannot be observed are not physical. Consequently, as long as the mass is an observable physical quantity, the shape of the string may not be physical, while physical quantities obey T-duality. Thus, physically,  $l_s$  is the shortest possible distance, which implies nonlocality. Our point (demonstrated by the discussion above, but valid also for T-duality of open strings) is that the assumption of orthodox interpretation of QM is the crucial assumption. Conversely, if quantities unobservable by the standard QM rules are still physical at some more fundamental level, then, at this more fundamental level, T-duality may not be a universal symmetry. In QM, such hypothetical unobservable degrees of freedom are referred to as hidden variables. The Bell theorem [10] shows that any hidden variables compatible with the statistical predictions of QM must necessarily be nonlocal. In the quantum theory of particles and fields, nonlocality is usually considered unacceptable. Consequently, the Bell theorem is usually considered a strong argument against the existence of hidden variables. However, in string theory, nonlocality seems to be unavoidable, so nonlocality can no longer be viewed as an argument against hidden variables. Quite the contrary, the existence of string nonlocalities, which are still far from being completely understood, may be viewed as an indication that the principles of QM itself should be reformulated in a nonlocal manner, perhaps by using a nonlocal hidden-variable formulation of QM. The results of [24] suggest that this reformulation should be in terms of the Bohmian hidden variables. In this paper we discuss how the Bohmian formulation of strings transcends T-duality at the fundamental level and introduces a different, clearer, form of nonlocality manifested as a nonlocal communication between different parts of the string.

### 3 Bohmian hidden variables for strings

The formulation of Bohmian interpretation of particles and fields [12–19] is usually based on canonical quantization in the Schrödinger picture. Accordingly, in this paper, we use the same picture, such that the target spacetime covariance is manifest, whereas the world-sheet covariance is not. (For the canonical quantization of strings in which the world-sheet covariance is also manifest, see [24].) For simplicity, we study only bosonic strings, but fermionic degrees of freedom can also be interpreted in a Bohmian deterministic manner [19], which allows one to generalize the results of this paper to superstrings as well.

The Hamiltonian operator of a bosonic string is given by

$$\hat{H} = - \int d\sigma \frac{1}{2} \left[ \hat{P}^\alpha(\sigma) \hat{P}_\alpha(\sigma) + \frac{\partial X^\alpha(\sigma)}{\partial \sigma} \frac{\partial X_\alpha(\sigma)}{\partial \sigma} \right], \quad (4)$$

where

$$\hat{P}_\alpha(\sigma) = \frac{\delta}{\delta X^\alpha(\sigma)}, \quad (5)$$

$\alpha = 0, 1, \dots, 25$  denotes the target spacetime indices, the signature of the flat spacetime metric is  $(+, -, \dots, -)$ ,  $\sigma$  is the affine parameter along the string, and we use units  $\hbar = c = \alpha' = 1$ . Our results will not depend on whether the string is closed or open, so we do not specify the boundary conditions. The quantum state of the string  $\Psi[X(\sigma)]$  (where  $X = \{X^0, \dots, X^{25}\}$ ) is a functional of the string configuration described by the functions  $X^\alpha(\sigma)$ . The state satisfies the Hamiltonian constraint

$$(\hat{H} - a)\Psi = 0, \quad (6)$$

where  $a = 1$  when normal ordering is chosen. The quantity  $|\Psi[X(\sigma)]|^2$  can be interpreted, at least formally, as the probability density for the string coordinates to be equal to  $X^\alpha(\sigma)$ . (The problem of normalization can be solved by bounding the spectrum of the possible values of  $X^0$  or by fixing  $X^0 = \tau$ . However, these technical subtleties will not influence our main conclusions; so, for simplicity, we ignore them in the rest of the analysis.) Owing to the Hamiltonian constraint, this probability density does not depend on the world-sheet time  $\tau$ . By writing

$$\Psi = R e^{iS}, \quad (7)$$

where  $R$  and  $S$  are real functionals, one finds that the complex equation (6) is equivalent to the set of two real equations

$$- \int d\sigma \frac{1}{2} \left[ \frac{\delta S}{\delta X^\alpha(\sigma)} \frac{\delta S}{\delta X_\alpha(\sigma)} + \frac{\partial X^\alpha(\sigma)}{\partial \sigma} \frac{\partial X_\alpha(\sigma)}{\partial \sigma} \right] + Q - a = 0, \quad (8)$$

$$\int d\sigma \frac{\delta}{\delta X^\alpha(\sigma)} \left[ R^2 \frac{\delta S}{\delta X_\alpha(\sigma)} \right] = 0, \quad (9)$$

where

$$Q[X(\sigma)] = \int d\sigma \frac{1}{2R} \frac{\delta^2 R}{\delta X^\alpha(\sigma) \delta X_\alpha(\sigma)}. \quad (10)$$

The classical Hamiltonian constraint reads  $H = 0$ , so  $a = 0$  in the classical limit. By restoring units in which  $\hbar \neq 1$ , one finds that the right-hand side of (10) attains the additional prefactor  $\hbar^2$ . This shows that the classical limit corresponds to  $Q = a = 0$ . In this limit, (8) takes the form of the classical Hamilton–Jacobi equation. However, the classical Hamilton–Jacobi equation should be supplemented by the deterministic equation of motion of the string:

$$-\frac{\partial X_\alpha(\tau, \sigma)}{\partial \tau} = \frac{\delta S}{\delta X^\alpha(\sigma)} \Big|_{X(\sigma)=X(\tau, \sigma)}. \quad (11)$$

The Bohmian interpretation consists in the assumption that (11) is valid even in the quantum case with  $Q \neq 0$ . According to this interpretation, the string has an objective and deterministic evolution  $X^\alpha(\tau, \sigma)$ , even when this quantity is not measured. All quantum uncertainties are an artifact of the ignorance of the actual initial conditions  $X^\alpha(\sigma)$  at some initial time  $\tau$ . If in a statistical ensemble of string configurations each possible string configuration  $X(\sigma)$  has the probability  $|\Psi[X(\sigma)]|^2$  at some *initial* time, then (11) and (9) imply that each possible string configuration  $X(\sigma)$  has the probability  $|\Psi[X(\sigma)]|^2$  at *any* time  $\tau$ . This explains why such a deterministic evolution of strings is in agreement with the statistical predictions of the orthodox interpretation of quantum strings. (For more details on the agreement between the statistical predictions of the Bohmian and the orthodox interpretation, we refer the reader to [12, 13, 16, 18].) Equation (8) is interpreted as the quantum Hamilton–Jacobi equation. The quantity (10) is referred to as the *quantum potential*. From (11) and (8) one can derive the quantum equation of motion:

$$\frac{\partial^2 X^\alpha(\tau, \sigma)}{\partial \tau^2} - \frac{\partial^2 X^\alpha(\tau, \sigma)}{\partial \sigma^2} = \frac{\delta Q}{\delta X_\alpha(\sigma)} \Big|_{X(\sigma)=X(\tau, \sigma)}. \quad (12)$$

The right-hand side represents the quantum modification of the classical string equation of motion. This modification represents the quantum force given by the functional gradient of the quantum potential.

We stress that by postulating the new Bohmian equation of motion (11) we do *not* modify any of the standard equations of string theory that define the physical states, such as (6). In particular, the mass spectrum (3) of string states is the same as that in the usual formulation of string theory, which does not require any new verification. Therefore, T-duality at the observable level appears in exactly the same way as that in the usual formulation of string theory. Nevertheless, the Bohmian interpretation offers a different interpretation of that standard result.

The string coordinates  $X^\alpha(\tau, \sigma)$  are the hidden variables. Their existence means that the string has a definite position and shape at all length scales even when it is not measured. These hidden variables are not invariant under a T-duality transformation. Consequently, T-duality is broken at the fundamental hidden-variable level and  $l_s$  is not the minimal possible length. In fact, at the kinematic level, such a quantum string does not differ from the classical one. However, it does not mean that locality of classical strings is also restored at the quantum

level. Instead, as seen from (12), nonlocality reappears at the dynamic level. Namely, at a given time  $\tau$ , the quantum force on the string at the point  $\sigma$ , in general, is not merely a function of  $X^\alpha(\sigma)$ , but a functional of the *whole* functions  $X^\alpha(\sigma')$  at *all* points  $\sigma'$ . In other words, different parts of the string may communicate in a nonlocal manner. To better understand the physical origin of this nonlocality, consider a state  $\Psi$  that has the form of a local product:

$$\Psi[X(\sigma)] = \prod_\sigma \psi_\sigma(X(\sigma)). \quad (13)$$

In this case,  $R$  also takes a similar local-product form, so (10) implies that the quantum potential is an integral of the form

$$Q[X(\sigma)] = \int d\sigma \mathcal{Q}(X(\sigma)), \quad (14)$$

where  $\mathcal{Q}(X(\sigma))$  is a local quantum-potential density. In this case, the quantum force on the string at the point  $\sigma$  is given by

$$\frac{\delta Q}{\delta X_\alpha(\sigma)} = \frac{\partial \mathcal{Q}(X(\sigma))}{\partial X_\alpha(\sigma)} \equiv \mathcal{F}^\alpha(X(\sigma)). \quad (15)$$

We see that this force is local, as it depends only on  $X(\sigma)$ . However, in general, the quantum state does *not* have a local-product form (13). (Even the ground state does not have such a form.) Consequently, unlike (14), the quantum potential cannot be written as an integral over a local quantum-potential density  $\mathcal{Q}(X(\sigma))$ , which implies that, unlike (15), the quantum force on the string at the point  $\sigma$  does not depend only on  $X(\sigma)$ . As states that do not have the local-product form (13) exhibit quantum entanglement, we see that the nonlocal communication between different parts of the string is a direct consequence of quantum entanglement between them. Since this nonlocality is realized only on the dynamic level, while kinematics is still local and classical, this form of nonlocality does not require a radical revision of the concept of continuous geometry. Nevertheless, by construction, these nonlocal interactions have exactly the form needed for different parts of the string to conspire so that the observable (i.e., not hidden) properties of the string obey T-duality.

From a practical point of view, it is fair to note that the Bohmian hidden-variable formulation of strings does not directly lead to new observable effects in perturbative string theory. Nevertheless, we believe that such a formulation might radically modify the existing attempts to find a satisfying *nonperturbative* formulation of string theory, as the existing attempts often significantly deviate from the original picture of strings as world-sheets in spacetime, whereas the Bohmian interpretation suggests that such a picture should be taken seriously.

To summarize, the Bohmian hidden-variable formulation of strings, suggested by the results of [24], breaks T-duality at the fundamental level and introduces a new form of nonlocality into string theory, manifested as a non-local interaction between different parts of the string. Such a form of nonlocality does not demand a modification of the usual picture of continuous geometry.

*Acknowledgements.* This work was supported by the Ministry of Science and Technology of the Republic of Croatia.

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